

Week 07 Worksheet: Cauchy-Euler equations

To receive credit, hand in as many solved practice problems as time permits. Try unfinished problems at home. Solution of this worksheet will be made available on the website.

1. (Demonstration) Solve

(a) $x^2y'' + xy' - y = 0; \quad y(0) = 0, \quad y(1) = 1$

(b) $x^2y'' - xy' + (1 + \pi^2/4)y = 0; \quad y(1) = 1, \quad y(e) = e$

(c) $x^2y'' + 3xy' + y = 0; \quad y(1) = 1, \quad y(e) = 1$

2. (Practice) Solve

(a) $x^2y'' - 2xy' + 2y = 0, \quad y'(0) = 1, \quad y(1) = 0$

(b) $2x^2y'' - xy' + y = 0, \quad y(1) = 0, \quad y(4) = 1$

3. (Practice) Solve

(a) $x^2y'' - xy' + (1 + \pi^2)y = 0, \quad y(1) = 1, \quad y(\sqrt{e}) = \sqrt{e}$

(b) $x^2y'' + 3xy' + (1 + \pi^2)y = 0, \quad y(1) = 1, \quad y(\sqrt{e}) = \sqrt{e}$

4. (Practice) Solve

(a) $x^2y'' - xy' + y = 0, \quad y(1) = 1, \quad y'(1) = 0$

(b) $4x^2y'' + y = 0, \quad y(1) = 1, \quad y(e) = 0$