

Worksheet 6: Series solutions

To receive credit, hand in as many solved practice problems as time permits. Try unfinished problems at home. Solution of this worksheet will be made available on the website.

1. **(Demonstration)** Find two independent power series solutions to the following differential equation, computing solutions up to x^6 .

$$y'' + xy' + y = 0.$$

2. **(Practice)** Find two independent power series solutions to the following differential equation, where the highest power of x to be computed is specified:

(a) $y'' + xy' - y = 0$ (x^6)

(b) $y'' + y' + xy = 0$ (x^5)

3. **(Practice)** The Hermite equation is given by

$$y'' - 2xy' + 2\lambda y = 0,$$

where λ is a constant.

- (a) Find the first three terms in each of two independent power series solutions.
- (b) If $\lambda = n$ is a nonnegative integer, then one of the power series solutions will become a polynomial. Find the polynomial solutions for $n = 0, 1, 2, 3$.
- (c) The Hermite polynomials $H_n(x)$ are the polynomial solutions of the Hermite equation normalized so that the coefficient of x^n is 2^n . Find $H_0(x)$, $H_1(x)$, $H_2(x)$, $H_3(x)$.