Worksheet 5: The Laplace transform

To receive credit, hand in as many solved practice problems as time permits. Try unfinished problems at home. Solution of this worksheet will be made available on the website.

1. **(Demonstration)** Model how to solve a standard inhomogeneous ode using the Laplace transform technique:

   $$\ddot{x} + 2\dot{x} + 5x = e^{-t}, \quad x(0) = 0,\ \dot{x}(0) = 0.$$

2. **(Demonstration)** Model how to solve an ode with a piecewise-continuous inhomogeneous term:

   $$\ddot{x} + 3\dot{x} + 2x = \begin{cases} t & \text{if } 0 \le t < 1, \\ 1 & \text{if } t \ge 1; \end{cases} \quad x(0) = 0,\ \dot{x}(0) = 0.$$

3. **(Demonstration)** Model how to solve an ode with a Dirace delta-function inhomogeneous term:

   $$\ddot{x} + 2\dot{x} + x = \delta(t-1), \quad x(0) = 1,\ \dot{x}(0) = 0.$$

4. **(Practice)** Solve the following inhomogeneous odes using the Laplace transform technique:

   (a) $\ddot{x} + 2\dot{x} + 5x = e^{-2t}, \quad x(0) = 0,\ \dot{x}(0) = 0$

   (b) $\ddot{x} + 3\dot{x} + 2x = \begin{cases} 1-t & \text{if } 0 \le t < 1, \\ 0 & \text{if } t \ge 1; \end{cases} \quad x(0) = 0,\ \dot{x}(0) = 0$

   (c) $\ddot{x} + 2\dot{x} + x = \delta(t-1), \quad x(0) = 0,\ \dot{x}(0) = 1$