Week 03 Worksheet: Second-order homogeneous odes

To receive credit, hand in as many solved practice problems as time permits. Try unfinished problems at home. Solution of this worksheet will be made available on the website.

1. **(Demonstration)** Model the case of two real roots by solving for $x = x(t)$:

   $$\ddot{x} + 4\dot{x} + 3x = 0; \quad x(0) = 1,\ \dot{x}(0) = 1.$$

2. **(Demonstration)** Model the case of complex conjugate roots by solving for $x = x(t)$:

   $$\ddot{x} - 2\dot{x} + 5x = 0; \quad x(0) = 1,\ \dot{x}(0) = 1.$$

3. **(Demonstration)** Model the case of degenerate real roots by solving for $x = x(t)$:

   $$\ddot{x} + 4\dot{x} + 4x = 0; \quad x(0) = 1,\ \dot{x}(0) = 1.$$

4. **(Practice)** Solve for $x = x(t)$:
   (a) $\ddot{x} + 3\dot{x} + 2x = 0, \quad x(0) = 0,\ \dot{x}(0) = 1$
   (b) $\ddot{x} - 3\dot{x} + 2x = 0, \quad x(0) = 1,\ \dot{x}(0) = 0$
   (c) $\ddot{x} - 2\dot{x} + 2x = 0, \quad x(0) = 1,\ \dot{x}(0) = 0$
   (d) $\ddot{x} + 2\dot{x} + 2x = 0, \quad x(0) = 0,\ \dot{x}(0) = 1$
   (e) $\ddot{x} + 2\dot{x} + x = 0, \quad x(0) = 1,\ \dot{x}(0) = 0$
   (f) $\ddot{x} - 2\dot{x} + x = 0, \quad x(0) = 0,\ \dot{x}(0) = 1$

5. **(Practice)** Solve the initial value problem $\ddot{x} - \dot{x} - 2x = 0, \quad x(0) = \alpha,\ \dot{x}(0) = 2$ and determine the constant α so that the solution tends to zero as $t \to \infty$.

6. **(Practice)** Solve the initial value problem $4\ddot{x} - x = 0, \quad x(0) = 2,\ \dot{x}(0) = \beta$ and determine the constant β so that the solution tends to zero as $t \to \infty$.

7. **(Practice)** Find a differential equation whose general solution is given by $x = c_1 e^{2t} + c_2 e^{-3t}$, where c_1, c_2 are constants.