Week 01 Worksheet: Complex numbers and separable first-order equations

To receive credit, hand in as many solved practice problems as time permits. Complete all unfinished problems at home. Solution of this worksheet will be made available on the website.

1. **(Demonstration)** Complex number problems of various sorts.

2. **(Demonstration)** Model the following first-order separable ode for $y = y(x)$:

$$(1+y)y' = x, \qquad y(0) = 0.$$

3. **(Practice)** Write as a complex number $z = x + iy$, where x and y are real.

   (a) $\dfrac{1+3i}{3-2i}$

   (b) $\dfrac{1}{1+i} + \dfrac{1}{1-i}$

   (c) $\dfrac{-1-2i}{-4+3i}$

   (d) $-(7-i)(-4-2i)(2-i)$

4. **(Practice)** Convert to polar form $z = r \exp(i\theta)$.

   (a) $1 + \sqrt{3}i$

   (b) $(\sqrt{2} + \sqrt{2}i)^7$

5. **(Practice)** Solve for x and y, where x and y are real.
   $$2y + ix = 4 + x - i$$

6. **(Practice)** With x real, find the real and the imaginary parts
   $$\exp\big((5+12i)x\big)$$

7. **(Practice)** Solve separable odes for $y = y(x)$.

   (a) $y' = \sqrt{xy}, \quad y(1) = 0$

   (b) $y^2 - xy' = 0, \quad y(1) = 1$

   (c) $e^{x-y}y' + e^{y-x} = 0, \quad y(0) = 0$

   (d) $y' + (\sin x)y = 0, \quad y(\pi/2) = 1$

   (e) $y' = y(1-y), \quad y(0) = y_0 \quad (y_0 > 0)$