(c) Use part (b) to solve the same problem with the boundary data $h(x)$, where $h(x)$ is any step function. That is,

$$h(x) = c_j \quad \text{for } a_{j-1} < x < a_j \quad \text{for } 1 \le j \le n,$$

where $-\infty = a_0 < a_1 < \cdots < a_{n-1} < a_n = \infty$ and the c_j are constants.

9. Find the Green's function for the tilted half-space $\{(x, y, z): ax + by + cz > 0\}$. (*Hint:* Either do it from scratch by reflecting across the tilted plane, or change variables in the double integral (3) using a linear transformation.)
10. Verify the formula (11) for $G(\mathbf{x}, \mathbf{0})$, the Green's function with its second argument at the center of the sphere.
11. (a) Verify that (18) is the Green's function for the disk.
    (b) Use it to recover the Poisson formula.
12. Find the potential of the electrostatic field due to a point charge located outside a grounded sphere. (*Hint:* This is just the Green's function for the exterior of the sphere. Find it by the method of reflection.)
13. Find the Green's function for the half-ball $D = \{x^2 + y^2 + z^2 < a^2, z > 0\}$. (*Hint:* The easiest method is to use the solution for the whole ball and reflect it across the plane.)
14. Do the same for the eighth of a ball

$$D = \{x^2 + y^2 + z^2 < a^2,\ x > 0,\ y > 0,\ z > 0\}.$$

15. (a) Show that if $v(x, y)$ is harmonic, so is $u(x, y) = v(x^2 - y^2, 2xy)$.
    (b) Show that the transformation $(x, y) \longmapsto (x^2 - y^2, 2xy)$ maps the first quadrant onto the half-plane $\{y > 0\}$. (*Hint:* Use either polar coordinates or complex variables.)
16. Use Exercises 15 and 7 to find the harmonic function $u(x, y)$ in the first quadrant that has the boundary values $u(x, 0) = A$, $u(0, y) = B$, where A and B are constants. (*Hint:* $u(x, 0) = v(x^2, 0)$, etc.)
17. (a) Find the Green's function for the quadrant

$$Q = \{(x, y): x > 0, y > 0\}.$$

(*Hint:* Either use the method of reflection or reduce to the half-plane problem by the transformation in Exercise 15.)

(b) Use your answer in part (a) to solve the Dirichlet problem

$$u_{xx} + u_{yy} = 0 \text{ in } Q, \quad u(0, y) = g(y) \text{ for } y > 0,$$
$$u(x, 0) = h(x) \text{ for } x > 0.$$

18. (a) Find the Green's function for the octant $\mathcal{O} = \{(x, y, z): x > 0, y > 0, z > 0\}$. (*Hint:* Use the method of reflection.)