2. Solve $u_{xx} + u_{yy} = 0$ in the disk $r < a$ with the boundary condition

$$\frac{\partial u}{\partial r} - hu = f(\theta),$$

where $f(\theta)$ is an arbitrary function. Write the answer in terms of the Fourier coefficients of $f(\theta)$.

3. Determine the coefficients in the annulus problem of the text.

4. Derive Poisson's formula (9) for the exterior of a circle.

5. (a) Find the steady-state temperature distribution inside an annular plate $\{1 < r < 2\}$, whose outer edge ($r = 2$) is insulated, and on whose inner edge ($r = 1$) the temperature is maintained as $\sin^2 \theta$. (Find explicitly all the coefficients, etc.)
   (b) Same, except $u = 0$ on the outer edge.

6. Find the harmonic function u in the semidisk $\{r < 1, 0 < \theta < \pi\}$ with u vanishing on the diameter ($\theta = 0, \pi$) and

$$u = \pi \sin\theta - \sin 2\theta \quad \text{on } r = 1.$$

7. Solve the problem $u_{xx} + u_{yy} = 0$ in D, with $u = 0$ on the two straight sides, and $u = h(\theta)$ on the arc, where D is the wedge of Figure 1, that is, a sector of angle β cut out of a disk of radius a. Write the solution as a series, but don't attempt to sum it.

8. An annular plate with inner radius a and outer radius b is held at temperature B at its outer boundary and satisfies the boundary condition $\partial u/\partial r = A$ at its inner boundary, where A and B are constants. Find the temperature if it is at a steady state. (*Hint:* It satisfies the two-dimensional Laplace equation and depends only on r.)

9. Solve $u_{xx} + u_{yy} = 0$ in the wedge $r < a, 0 < \theta < \beta$ with the BCs

$$u = \theta \quad \text{on } r = a, \qquad u = 0 \quad \text{on } \theta = 0, \qquad \text{and} \qquad u = \beta \quad \text{on } \theta = \beta.$$

(*Hint:* Look for a function independent of r.)

10. Solve $u_{xx} + u_{yy} = 0$ in the quarter-disk $\{x^2 + y^2 < a^2, x > 0, y > 0\}$ with the following BCs:

$$u = 0 \quad \text{on } x = 0 \text{ and on } y = 0 \quad \text{and} \quad \frac{\partial u}{\partial r} = 1 \quad \text{on } r = a.$$

Write the answer as an infinite series and write the first two nonzero terms explicitly.

11. Prove the uniqueness of the Robin problem

$$\Delta u = f \quad \text{in } D, \qquad \frac{\partial u}{\partial n} + au = h \quad \text{on bdy } D,$$

where D is any domain in three dimensions and where a is a positive constant.