(a) Find the full Fourier series of $\phi(x)$ in the interval $(-1, 1)$.
(b) Find the first three nonzero terms explicitly.
(c) Does it converge in the mean square sense?
(d) Does it converge pointwise?
(e) Does it converge uniformly to $\phi(x)$ in the interval $(-1, 1)$?

8. Consider the Fourier sine series of each of the following functions. In this exercise do not compute the coefficients but use the general convergence theorems (Theorems 2, 3, and 4) to discuss the convergence of each of the series in the pointwise, uniform, and L^2 senses.
(a) $f(x) = x^3$ on $(0, l)$.
(b) $f(x) = lx - x^2$ on $(0, l)$.
(c) $f(x) = x^{-2}$ on $(0, l)$.

9. Let $f(x)$ be a function on $(-l, l)$ that has a continuous derivative and satisfies the periodic BCs. Let a_n and b_n be the Fourier coefficients of $f(x)$, and let a_n' and b_n' be the Fourier coefficients of its derivative $f'(x)$. Show that

$$a_n' = \frac{n\pi b_n}{l} \qquad \text{and} \qquad b_n' = \frac{-n\pi a_n}{l} \qquad \text{for } n \neq 0.$$

(*Hint:* Write the formulas for a_n' and b_n' and integrate by parts.) This means that the Fourier series of $f'(x)$ is what you'd obtain as if you differentiated term by term. It does not mean that the differentiated series converges.

10. Deduce from Exercise 9 that there is a constant k so that

$$|a_n| + |b_n| \leq \frac{k}{n} \qquad \text{for all } n.$$

11. (*Term by term integration*)
(a) If $f(x)$ is a piecewise continuous function in $[-l, l]$, show that its indefinite integral $F(x) = \int_{-l}^{x} f(s)\,ds$ has a full Fourier series that converges pointwise.
(b) Write this convergent series for $f(x)$ explicitly in terms of the Fourier coefficients a_0, a_n, b_n of $f(x)$.
(*Hint:* Apply a convergence theorem. Write the formulas for the coefficients and integrate by parts.)

12. Start with the Fourier sine series of $f(x) = x$ on the interval $(0, l)$. Apply Parseval's equality. Find the sum $\Sigma_{n=1}^{\infty} 1/n^2$.

13. Start with the Fourier cosine series of $f(x) = x^2$ on the interval $(0, l)$. Apply Parseval's equality. Find the sum $\Sigma_{n=1}^{\infty} 1/n^4$.

14. Find the sum $\Sigma_{n=1}^{\infty} 1/n^6$.

15. Let $\phi(x) \equiv 1$ for $0 < x < \pi$. Expand

$$1 = \sum_{n=0}^{\infty} B_n \cos\left[\left(n + \tfrac{1}{2}\right)x\right].$$