is the cornerstone of the theory of relativity. It means that a signal located at the position x_0 at the instant t_0 cannot move faster than the speed of light. The domain of influence of (x_0, t_0) consists of all the points that can be reached by a signal of speed c starting from the point x_0 at the time t_0. It turns out that the solutions of the *three*-dimensional wave equation always travel at speeds exactly equal to c and never slower. Therefore, the causality principle is sharper in three dimensions than in one. This sharp form is called *Huygens's principle* (see Chapter 9).

Flatland is an imaginary two-dimensional world. You can think of yourself as a waterbug confined to the surface of a pond. You wouldn't want to live there because Huygens's principle is not valid in two dimensions (see Section 9.2). Each sound you make would automatically mix with the "echoes" of your previous sounds. And each view would be mixed fuzzily with the previous views. Three is the best of all possible dimensions.

EXERCISES

1. Use the energy conservation of the wave equation to prove that the only solution with $\phi \equiv 0$ and $\psi \equiv 0$ is $u \equiv 0$. (*Hint:* Use the first vanishing theorem in Section A.1.)
2. For a solution $u(x, t)$ of the wave equation with $\rho = T = c = 1$, the energy density is defined as $e = \frac{1}{2}(u_t^2 + u_x^2)$ and the momentum density as $p = u_t u_x$.
   (a) Show that $\partial e/\partial t = \partial p/\partial x$ and $\partial p/\partial t = \partial e/\partial x$.
   (b) Show that both $e(x, t)$ and $p(x, t)$ also satisfy the wave equation.
3. Show that the wave equation has the following invariance properties.
   (a) Any translate $u(x - y, t)$, where y is fixed, is also a solution.
   (b) Any derivative, say u_x, of a solution is also a solution.
   (c) The dilated function $u(ax, at)$ is also a solution, for any constant a.
4. If $u(x, t)$ satisfies the wave equation $u_{tt} = u_{xx}$, prove the identity

   $$u(x + h, t + k) + u(x - h, t - k) = u(x + k, t + h) + u(x - k, t - h)$$

   for all x, t, h, and k. Sketch the quadrilateral Q whose vertices are the arguments in the identity.
5. For the *damped* string, equation (1.3.3), show that the energy decreases.
6. Prove that, among all possible dimensions, only in three dimensions can one have distortionless spherical wave propagation with attenuation. This means the following. A spherical wave in n-dimensional space satisfies the PDE

   $$u_{tt} = c^2\left(u_{rr} + \frac{n-1}{r}u_r\right),$$

   where r is the spherical coordinate. Consider such a wave that has the special form $u(r, t) = \alpha(r) f(t - \beta(r))$, where $\alpha(r)$ is called the