

MATH 2352 Problem Sheet 08

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[Problems] 6.4: 10, 12; 6.5: 3, 7; 6.6: 1(c), 7, 10, 17, 20;

6.4 - 10. For

$$y'' + y' + \frac{5}{4}y = g(t);$$

$$y(0) = 1,$$

$$y'(0) = 0;$$

$$g(t) = \begin{cases} \sin t, & 0 \leq t < \pi \\ 0, & t \geq \pi \end{cases}.$$

- (a) Find the solution of the given initial value problem.
(b) Draw the graphs of the solution and of the forcing function; explain how they are related.

6.4 - 12. For

$$y^{(4)} - y = u_1(t) - u_3(t);$$

$$y(0) = 0,$$

$$y'(0) = 0,$$

$$y''(0) = 0,$$

$$y'''(0) = 0;$$

$$u_c(t) = \begin{cases} 0, & t < c, \\ 1, & t \geq c. \end{cases}$$

- (a) Find the solution of the given initial value problem.
(b) Draw the graphs of the solution and of the forcing function; explain how they are related.

6.5 - 3. For

$$y'' + 3y' + 2y = \delta(t - \pi);$$

$$y(0) = 1,$$

$$y'(0) = 1.$$

(a) Find the solution of the given initial value problem.

(b) Draw a graph of the solution.

6.5 - 7. For

$$y'' + y = \delta(t - 2\pi) \cos t;$$

$$y(0) = 1,$$

$$y'(0) = 1;$$

(a) Find the solution of the given initial value problem.

(b) Draw a graph of the solution.

6.6 - 1 (c). Prove the associative property of the convolution integral:

$$f \star (g \star h) = (f \star g) \star h.$$

6.6 - 7. Find the Laplace transform of

$$f(t) = \int_0^t \sin(t - \tau) \cos 3\tau d\tau.$$

6.6 - 10. Find the inverse Laplace transform of

$$F(s) = \frac{1}{(s+1)^3(s^2+4)}$$

by using the convolution theorem.

6.6 - 17. Express the solution of the initial value problem

$$y'' + 4y' + 4y = g(t);$$

$$y(0) = 1,$$

$$y'(0) = -2;$$

in terms of a convolution integral.

6.6 - 20. Express the solution of the initial value problem

$$y^{(4)} + 5y'' + 4y = g(t);$$

$$y(0) = 1,$$

$$y'(0) = 0,$$

$$y''(0) = 0,$$

$$y'''(0) = 0;$$

in terms of a convolution integral.