MATH 2352 Problem Sheet 07

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[Problems] 6.1: 5(c), 14; 6.2: 2, 8, 14, 21; 6.3: 9, 11, 13, 20, 23, 25;

6.1 - 5. (c) Find the Laplace transform of each of the following functions

$$f(t) = t^n,$$

where n is a positive integer.

6.1 - 14. Recall that $\cos bt = (e^{ibt} + e^{-ibt})/2$ and that $\sin(bt) = (e^{ibt} + e^{-ibt})/2i$. Find the Laplace transform of the given function

$$f(t) = e^{at} \cos bt,$$

where a and b are real constants. Assume that the necessary elementary integration formulas extend to this case.

6.2 - 2. Find the inverse Laplace transform of the given function

$$F(s) = \frac{5}{(s-1)^3}.$$

6.2 - 8. Find the inverse Laplace transform of the given function

$$F(s) = \frac{8s^2 - 6s + 12}{s(s^2+4)}.$$

6.2 - 14. Use Laplace transform to solve the given initial value problem

$$\begin{aligned} y'' - 4y' + 4y &= 0, \\ y(0) &= 1, \\ y'(0) &= 3. \end{aligned}$$

6.2 - 21. Use Laplace transform to solve the given initial value problem

$$\begin{aligned} y'' - 2y' + 2y &= \cos t, \\ y(0) &= 1, \\ y'(0) &= 1. \end{aligned}$$

6.3 - 9. For

$$f(t) = \begin{cases} 1, & 0 \leqslant t < 2, \\ e^{-2(t-2)}, & t \geqslant 2. \end{cases}$$

(a) Sketch the graph of the given function.

(b) Express $f(t)$ in terms of the unit step function $u_c(t)$.

6.3 - 11. For

$$f(t) = \begin{cases} t, & 0 \leqslant t < 1, \\ t-1, & 1 \leqslant t < 2, \\ t-2, & 2 \leqslant t < 3 \\ 0, & t \geqslant 3. \end{cases}$$

(a) Sketch the graph of the given function.

(b) Express $f(t)$ in terms of the unit step function $u_c(t)$.

6.3 - 13. Find the Laplace transform of the given function.

$$f(t) = \begin{cases} 0, & t < 2, \\ (t-2)^3, & t \geqslant 2. \end{cases}$$

6.3 - 20. Find the inverse Laplace transform of the given function.

$$F(s) = \frac{e^{-3s}}{s^2 + s - 2}.$$

6.3 - 23. Find the inverse Laplace transform of the given function.

$$F(s) = \frac{(s-2)\,e^{-2s}}{s^2 - 4s + 3}.$$

6.3 - 25. Suppose that $F(s) = \mathcal{L}\{f(t)\}$ exists for $s > a \geqslant 0$.

(a) Show that if c is a positive constant, then

$$\mathcal{L}\{f(ct)\} = \frac{1}{c} F\left(\frac{s}{c}\right), \quad s > ca.$$

(b) Show that if k is a positive constant, then

$$\mathcal{L}^{-1}\{F(ks)\} = \frac{1}{k} f\left(\frac{t}{k}\right).$$

(c) Show that if a and b are constants with $a > 0$, then

$$\mathcal{L}^{-1}\{F(as+b)\} = \frac{1}{a} e^{-bt/a} f\left(\frac{t}{a}\right).$$