MATH 2352 Problem Sheet 05

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[Problems] 4.4: 5, 13; 5.1: 7, 13, 23, 25; 5.2: 9, 19; 5.4: 6, 19, 20, 35.

Happy Pie Day!

4.4 - 5. Use the method of variation of parameters to determin the general solution of the given differntial equation.

$$y''' - y'' + y' - y \;=\; 2e^{-t}\sin t.$$

4.4 - 13. Given that x, x^2 and $1/x$ are solutions of the homogeneous equation corresponding to

$$x^3 y''' + x^2 y'' - 2x\,y' + 2y \;=\; 2x^4, \quad x > 0,$$

determine a particular solution.

5.1 - 7. Determine the radius of convergence of the given power series.

$$S(x) \;=\; \sum_{n=1}^{\infty} \frac{(-1)^n n^2 (x+2)^n}{4^n}.$$

5.1 - 13. Determine the Taylor series about the point x_0 for the given function. Also determine the radius of convergence of the series.

$$\ln x, \quad x_0 = 1.$$

5.1 - 23. Rewrite the given expression as a sum whose generic term involves x^n.

$$S(x) \;=\; x^2 \sum_{n=1}^{\infty} n\,a_n\,x^{n-1} + \sum_{k=0}^{\infty} a_k\,x^k.$$

5.1 - 25. Rewrite the given expression as a sum whose generic term involves x^n.

$$S(x) \;=\; \sum_{m=2}^{\infty} m(m-1)a_m\,x^{m-2} + x^3 \sum_{k=1}^{\infty} k\,a_k\,x^{k-1}.$$

5.2 - 9. For

$$\begin{aligned}(1+x^2)y'' - 4\,x\,y' + 6y &= 0,\\ x_0 &= 0.\end{aligned}$$

(a) Seek power series solutions of the given differential equation about the given point x_0; find the recurrence relation.

(b) Find the first four terms in each of two solutions y_1 and y_2 (unless the series terminates sooner).

(c) By evaluating the Wronskian $W(y_1, y_2)(x_0)$, show that y_1 and y_2 form a fundamental set of solutions.

(d) If possible, find the general term in each solution.

5.2 - 19.

(a) By making the change of variable $x-1=t$ and assuming that y has a Taylor series in power of t, find two series solutions of

$$y'' + (x-1)^2 y' + (x^2-1)y \;=\; 0$$

in power series of $x-1$.

(b) Show that you obtain the same result by assuming that y has a Taylor series in power of $x-1$ and also expressing the coefficient x^2-1 in powers of $x-1$.

5.4 - 6. Determine the general solution of the given differential equation that is valid in any interval not including the singular point.

$$(x-1)^2 y'' + 8(x-1)y' + 12y \;=\; 0.$$

5.4 - 19. Find all singular points of the given equation and determine whether each one is regular or irregular.

$$x^2(1-x)y'' + (x-3)y' - 3x\,y \;=\; 0.$$

5.4 - 20. Find all singular points of the given equation and determine whether each one is regular or irregular.

$$x^2(1-x^2)y'' + (2/x)y' + 4y \;=\; 0.$$

5.4 - 35. Find all values of α for which all solutions of $x^2y'' + \alpha\, x\, y' + (5/2)y = 0$ approach zero as $x \to 0$.