MATH 2352 Problem Sheet 03

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Created on March 3, 2015

[Problems] 2.7: 16; 3.1: 4, 11; 3.3: 14, 19; 3.4: 6, 11; 3.2: 9, 10, 14, 25; 3.5: 6, 20;

2.7 - 16. Consider the initial value problem

$$y' = t^2 + y^2, \quad y(0) = 1.$$

Use Euler's method with $h = 0.1, 0.05, 0.025$, and 0.01 to explore the solution of this problem for $0 \leqslant t \leqslant 1$. What is your best estimate of the value of the solution at $t = 0.8$? At $t = 1$? Are your results consistent with the direction field in Problem 9?

3.1 - 4. Find the general solution of the given differential equation:

$$3\,y'' - 4\,y' + y = 0.$$

3.1 - 11. Find the solution of the given initial value problem. Sketch the graph of the solution and describe its behavior as t increases.

$$\begin{aligned} 12y'' - 7y' + y &= 0, \\ y(0) &= 4, \\ y'(0) &= 0. \end{aligned}$$

3.3 - 14. Find the general solution of given differential equation:

$$9y'' + 3y' - 2y = 0.$$

3.3 - 19. Find the solution of the given initial value problem. Sketch the graph of the solution and describe its behavior as t increases.

$$\begin{aligned} y'' - 2y' + 5y &= 0, \\ y\left(\frac{\pi}{2}\right) &= 0, \\ y'\left(\frac{\pi}{2}\right) &= 4. \end{aligned}$$

3.4 - 6. Find the general solution of given differential equation:

$$y'' - 10y' + 25y = 0.$$

3.4 - 11. Find the solution of the given initial value problem. Sketch the graph of the solution and describe its behavior as t increases.

$$\begin{aligned} 9y'' - 12y' + 4y &= 0, \\ y(0) &= 2, \\ y'(0) &= -2. \end{aligned}$$

3.2 - 9. Determin the longest interval in which the given initial value problem is certain to have a unique twice-differentiable solution. Do not attempt to find the solution.

$$\begin{aligned} t(t-4)y'' + 3ty' + 5y &= 2, \\ y(3) &= 0, \\ y'(3) &= -1. \end{aligned}$$

3.2 - 10. Determin the longest interval in which the given initial value problem is certain to have a unique twice-differentiable solution. Do not attempt to find the solution.

$$\begin{aligned} y'' + (\sin t)y' + 3(\ln|t|)y &= 0, \\ y(1) &= 3, \\ y'(1) &= 1. \end{aligned}$$

3.2 - 14. Verify that $y_1(t) = 1$ and $y_2(t) = t^{1/2}$ are solution of the differential equation $y\,y'' + (y')^2 = 0$ for $t > 0$. Then show that $y = c_1 + c_2\,t^{1/2}$ is not, in general, a solution of theis equation. Explain why this result does not contradict Theorem 3.2.2.

3.2 - 25. Verify that the functions y_1 and y_2 are solutions of the given differential equation. Do they constitute a fundamental set of solutions?

$$\begin{aligned} y'' - 4\,y' + 4y &= 0; \\ y_1(t) &= e^{2t}, \\ y_2(t) &= t\,e^{2t}. \end{aligned}$$

3.5 - 6. Find the general solution of given differential equation:

$$y'' + 2y' = 5 + 4\sin 2t.$$

3.5 - 20. Find the solution of the given initial value problem:

$$\begin{aligned} y'' + 2y' + 5y &= 4\,e^{-t}\cos 2t, \\ y(0) &= 0, \\ y'(0) &= 0. \end{aligned}$$