

MATH 2352 Problem Sheet 02

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[Problems] 2.2: 3, 7, 16(a,c); 2.3: 2, 9; 2.4: 4, 6, 13; 2.6: 7, 9

2.2 - 3. Solve the given differential equation

$$y' + y^2 \cos x = 0.$$

2.2 - 7. Solve the given differential equation

$$\frac{dy}{dx} = \frac{3x - e^{-x}}{2y + e^y}.$$

2.2 - 16. For the equation

$$y' = x(x^2 + 1)/4y^3, \quad y(0) = -1/\sqrt{2}.$$

(a) Find the solution of the given initial value problem in explicit form.

(c) Determine (at least approximately) the interval in which the solution is defined.

2.3 - 2. A tank initially contains 180 (L) of pure water. A mixture containing a concentration of γ (g/L) of salt enters the tank at a rate of 3 (L/min), and the well-stirred mixture leaves the tank at the same rate. Find an expression in terms of γ for the amount of salt in the tank at any time t . Also find the limiting amount of salt in the tank as $t \rightarrow \infty$.

2.3 - 9. A certain college graduate borrows \$9000 to buy a car. The lender charges interest at an annual rate of 8%. Assuming that interest is compounded continuously and that the borrower makes payments continuously at a constant annual rate k , determine the payment rate k that is required to pay off the loan in 3 years. Also determine how much interest is paid during the 3-year period.

2.4 - 4. Determine (without solving the problem) an interval in which the solution of the given initial value problem is certain to exist.

$$(16 - t^2) y' + 2t y = 3t^2, \quad y(-5) = 1.$$

2.4 - 6. Determine (without solving the problem) an interval in which the solution of the given initial value problem is certain to exist.

$$(\ln t) y' + y = \cot t, \quad y(3) = 3.$$

2.4 - 13. Solve the given initial value problem and determine how the interval in which the solution exists depends on the initial value y_0 .

$$y' = -2t/y, \quad y(0) = y_0.$$

2.6 - 7. Determine whether the equation is exact. If it is exact, find the solution.

$$(e^x \sin y - 3y \sin x) + (e^x \cos y + 3 \cos x) y' = 0.$$

2.6 - 9. Determine whether the equation is exact. If it is exact, find the solution.

$$(y e^{xy} \cos 2x - 2 e^{xy} \sin 2x + 2x) + (x e^{xy} \cos 2x - 3) y' = 0.$$