MATH 2111 Matrix Algebra and Applications — Week 10 Tutorial

1. Consider the polynomial space $\mathbb{P}$. Is the polynomial $p(t) = 1 - t^4$ a linear combination of polynomials in $S = \{1+t, t+t^2, t^2+t^3, t^3+t^4\}$?

2. Consider the function space V with a common domain $D = [0, \frac{\pi}{4}]$. Is the function $f(x) = \tan x$ a linear combination of functions in $S = \{\sin x, \cos x\}$?

3. Consider the function space V with a common domain $D = \mathbb{R}$. Is the function $f(x) = e^x$ a linear combination of functions in $S = \{\sin x, \cos x\}$?

4. Express $\operatorname{Nul} A$ as a span of a suitable set of vectors.

$$\text{(i) } A = \begin{bmatrix} 1 & 0 \\ 2 & 0 \end{bmatrix} \quad \text{(ii) } A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & -1 \end{bmatrix} \quad \text{(iii) } A = \begin{bmatrix} 1 & 2 & 3 & 1 & 6 \\ 2 & -1 & 1 & 1 & -1 \\ 1 & 1 & 2 & 1 & 3 \\ 2 & 1 & 3 & 1 & 5 \end{bmatrix}.$$

5. Check if $\mathbf{v} \in \operatorname{Col} A$ where $\mathbf{v}, A$ are given by:

$$\text{(i) } \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 2 & 3 \end{bmatrix} \quad \text{(ii) } \begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 & 3 & 1 \\ 2 & -5 & 1 \\ 3 & -2 & 2 \end{bmatrix} \quad \text{(iii) } \begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 & 1 & 0 & 4 & 5 \\ 2 & 5 & 1 & 1 & 3 \\ -2 & 3 & 1 & -7 & -7 \end{bmatrix}.$$

6. Let $\mathbf{v}, A$ be given as follows. Is $\mathbf{v} \in \operatorname{Row} A$?

$$\text{(i) } \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 & 2 & 0 \\ 2 & 1 & 1 \end{bmatrix} \quad \text{(ii) } \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 1 & 3 & 1 \\ 2 & -5 & 1 \\ 3 & -2 & 2 \end{bmatrix} \quad \text{(iii) } \begin{bmatrix} 5 \\ 7 \\ 2 \\ 10 \end{bmatrix}, \begin{bmatrix} 1 & 2 & -1 & 0 \\ 2 & -3 & 1 & 4 \\ -1 & 0 & 3 & 3 \end{bmatrix}.$$

7. In polynomial space, is $S = \{1+t, t+t^2, t^2+t^3, t^3+t^4, 1-t^4\}$ a linearly independent set?

8. In function space with domain $D = \mathbb{R}$, is $S = \{\cos t, \cos 2t\}$ a linearly independent set?

9. Determine if the following sets of vectors are bases for $\mathbb{R}^3$:

$$\text{(i) } \left\{ \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix} \right\} \quad \text{(ii) } \left\{ \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}, \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \right\} \quad \text{(iii) } \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \right\}.$$

Answers for checking:

1. Yes.
2. No.
3. No.
4. (i) $\text{Span}\{\begin{bmatrix} 0 \\ 1 \end{bmatrix}\}$ (ii) $\text{Span}\{\begin{bmatrix} -2 \\ 1 \\ 1 \end{bmatrix}\}$ (iii) $\text{Span}\{\begin{bmatrix} -1 \\ -1 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -2 \\ -3 \\ 0 \\ 2 \\ 1 \end{bmatrix}\}$.
5. (i) Yes (ii) No (iii) Yes.
6. (i) No (ii) No (iii) Yes.
7. No.
8. Yes.
9. (i) No (ii) No (iii) Yes.