

1. Write down the augmented matrices of the following systems:

$$(i) \begin{cases} x_1 + 2x_2 - 3x_3 = 0 \\ 2x_1 - x_2 + x_3 = 0 \\ 3x_1 + 3x_2 + x_3 = 0 \end{cases} \quad (ii) \begin{cases} x_1 + x_2 = 0 \\ x_2 + x_3 = 1 \\ x_3 + x_4 = 2 \end{cases} \quad (iii) \begin{cases} x_1 + x_5 = 1 \\ 3x_2 - x_4 = 4 \\ x_1 + 2x_3 = -3 \end{cases}$$

2. Write down the linear systems with the following augmented matrices:

$$(i) [1 \ 2 \ | \ 3] \quad (ii) \left[\begin{array}{c|c} 1 & 2 \\ 2 & 3 \end{array} \right] \quad (iii) \left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & -1 & 0 & 2 \\ 0 & 0 & 2 & 3 \end{array} \right] \quad (iv) \left[\begin{array}{cccc|c} 2 & 1 & 2 & 3 & 5 \\ 0 & -2 & 0 & 4 & 1 \end{array} \right]$$

3. Are the linear systems represented by the following two augmented matrices having the same solution sets?

$$(i) \left[\begin{array}{cc|c} 1 & 2 & 1 \\ 2 & 1 & 2 \end{array} \right], \left[\begin{array}{cc|c} 1 & 1 & 1 \\ 0 & 1 & 0 \end{array} \right] \quad (ii) \left[\begin{array}{c|c} 1 & 2 \\ 2 & 1 \end{array} \right], \left[\begin{array}{c|c} 0 & 2 \\ 2 & 0 \end{array} \right]$$

4. Is the following system consistent?

$$\begin{cases} x_1 + 3x_2 - 2x_3 + 5x_4 = 4 \\ 2x_1 + 8x_2 - x_3 + 9x_4 = 9 \\ 3x_1 + 5x_2 - 12x_3 + 17x_4 = 7 \end{cases}$$

5. Solve the following system and express the general solution in parametric form:

$$\begin{cases} x_1 + 2x_2 + 3x_3 + 2x_4 + 15x_5 = 1 \\ 2x_1 + 4x_2 - x_3 + 2x_4 + 8x_5 = 6 \\ 3x_1 + 6x_2 - x_3 + 3x_4 + 13x_5 = 8 \end{cases}$$

6. Perform the EROs in the given sequence to the matrix A .

$$(i) A = \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \quad 3r_1 + r_2$$

$$(ii) A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & 2 \end{bmatrix}, \quad \text{1st: } -r_1 + r_2, \text{ 2nd: } -r_2 + r_1$$

$$(iii) A = \begin{bmatrix} -1 & -1 & -1 \\ 1 & 2 & 3 \\ 1 & 3 & 5 \end{bmatrix}, \quad \text{1st: } r_1 + r_2, \text{ 2nd: } r_1 + r_3, \text{ 3rd: } -2r_2 + r_3$$

Answers for checking:

$$1. \text{ (i) } \left[\begin{array}{ccc|c} 1 & 2 & -3 & 0 \\ 2 & -1 & 1 & 0 \\ 3 & 3 & 1 & 0 \end{array} \right] \quad \text{(ii) } \left[\begin{array}{cccc|c} 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 2 \end{array} \right] \quad \text{(iii) } \left[\begin{array}{ccccc|c} 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 3 & 0 & -1 & 0 & 4 \\ 1 & 0 & 2 & 0 & 0 & -3 \end{array} \right].$$

$$2. \text{ (i) } \{ x_1 + 2x_2 = 3 \} \quad \text{(ii) } \begin{cases} x_1 = 2 \\ 2x_1 = 3 \end{cases} \quad \text{(iii) } \begin{cases} x_1 = 1 \\ -x_2 = 2 \\ 2x_3 = 3 \end{cases} \quad \text{(iv) } \begin{cases} 2x_1 + x_2 + 2x_3 + 3x_4 = 5 \\ -2x_2 + 4x_4 = 1 \end{cases}$$

3. (i) Yes (ii) Yes.

4. Inconsistent.

$$5. \begin{cases} x_1 = -3 - 2s - t \\ x_2 = s \\ x_3 = -2 - 2t \\ x_4 = 5 - 4t \\ x_5 = t \end{cases} \quad \text{where } s, t \text{ are free.}$$

$$6. \text{ (i) } \begin{bmatrix} 1 \\ 5 \end{bmatrix} \quad \text{(ii) } \begin{bmatrix} 0 & 3 & 0 \\ 1 & -1 & 1 \end{bmatrix} \quad \text{(iii) } \begin{bmatrix} -1 & -1 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}.$$