MATH 2111: Additional Explanations and Hints to Week 5 Tutorial

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Abstract

This document is provided as a part of supplemental materials for MATH 2111 Matrix Algebra and Applications (2015 autumn). Although it is written in the hope that it will be useful, nothing contained in this document represents the official views or policies of this course. Comments and suggestions are welcomed to be sent to the author (xweiaf@connect.ust.hk).

1 Problem 3

Proof. Let $A = [\alpha, \beta]$, where α, β are column vectors in $\mathbf{R}^2$. Then by the definition of orthogonal matrix, we have

$$\begin{aligned} Id &= A^T A \\ &= \begin{bmatrix} \alpha' \\ \beta' \end{bmatrix} \begin{bmatrix} \alpha & \beta \end{bmatrix} = \begin{bmatrix} \alpha'\alpha & \alpha'\beta \\ \beta'\alpha & \beta'\beta \end{bmatrix}. \end{aligned} \tag{1}$$

Therefore, we have $|\alpha| = |\beta| = 1$ from diagonal equations, and $\alpha \perp \beta$ from off-diagonal equations.

Then we can denote $\alpha = [\cos\theta, \sin\theta]'$, $\beta = [\cos(\theta \pm \frac{\pi}{2}), \sin(\theta \pm \frac{\pi}{2})]'$. Simplifying this gives us

$$A = \begin{bmatrix} \cos\theta & \sin\theta \\ \sin\theta & -\cos\theta \end{bmatrix}, \tag{2}$$

or

$$A = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}. \tag{3}$$

$\square$

Remark 1.1. The statement after "Therefore.." above is readily to be generalized to any orthogonal matrix.