MATH 2111: Additional Explanations and Hints to Week 5 Tutorial

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Abstract

This document is provided as a part of supplemental materials for MATH 2111 Matrix Algebra and Applications (2015 autumn). Although it is written in the hope that it will be useful, nothing contained in this document represents the official views or policies of this course. Comments and suggestions are welcomed to be sent to the author (xweiaf@connect.ust.hk).

1 Problem 7

Proof. Proof by induction.

Suppose A_n, B_n are $n \times n$ lower triangular matrices. $n = 1$ is trivial. If the statement holds for all positive integers no larger than n, then

$$\begin{aligned} A_{n+1}B_{n+1} &= \begin{bmatrix} A_n & 0 \\ \alpha' & a \end{bmatrix} \begin{bmatrix} B_n & 0 \\ \beta' & b \end{bmatrix} \\ &= \begin{bmatrix} A_nB_n & 0 \\ \alpha' B_n + a\beta' & ab \end{bmatrix}, \end{aligned} \tag{1}$$

which is lower triangular since A_nB_n is lower triangular by assumption. Therefore the statement also holds for $n + 1$. By induction, the statement shall hold for any positive integer. □

Remark 1.1. Actually, many operations on upper/lower triangular matrices preserve the shape:

- The sum of two upper/lower triangular matrices is upper/lower triangular.
- The product of two upper/lower triangular matrices is upper/lower triangular.
- The inverse of an invertible upper/lower triangular matrix is upper/lower triangular.
- The product of an upper/lower triangular matrix by a constant is an upper/lower triangular matrix.