MATH 2111: Additional Explanations and Hints to Week 3 Tutorial

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Abstract

This document is provided as a part of supplemental materials for MATH 2111 Matrix Algebra and Applications (2015 autumn). Although it is written in the hope that it will be useful, nothing contained in this document represents the official views or policies of this course. Comments and suggestions are welcomed to be sent to the author (xweiaf@connect.ust.hk).

1 Problem 6

Noting that A is in RRES, and classifying according to $rank(A)$, we have

1. $rank(A) = 0$, implying $A = 0$, counts 1.
2. $rank(A) = 1$, implying $A = [1, 1; 0, 0]$ or $A = [1, 0; 0, 0]$ or $A = [0, 1; 0, 0]$, counts 3.
3. $rank(A) = 2$, implying $A = [1, 0; 0, 1]$, counts 1.

In total, we count $1 + 3 + 1 = 5$ possibilities.

2 Problem 7

(Consider a linear system with p vaiables and q equations.)

Counter example when $p > q$ and inconsistent:

$$\begin{cases} x_1 + x_2 + x_3 = 1 \\ 2x_1 + 2x_2 + 2x_3 = 1 \end{cases}. \tag{1}$$

Counter example when $p < q$ and consistent:

$$\begin{cases} x_1 = 1 \\ 2x_1 = 2 \end{cases}. \tag{2}$$

Counter example when $p = q$ and the system has no solution:

$$\begin{cases} x_1 + x_2 = 1 \\ 2x_1 + 2x_2 = 1 \end{cases}. \tag{3}$$

Counter example when $p = q$ and the system has infinite solutions:

$$\begin{cases} x_1 + x_2 = 1 \\ 2x_1 + 2x_2 = 2 \end{cases}. \tag{4}$$

Counter example when $p \neq q$, the system being consistent, and have unique solution:

$$\begin{cases} x_1 + x_2 = 1 \\ x_1 - x_2 = 0 \\ 2x_1 + 2x_2 = 2 \end{cases}. \tag{5}$$

3 Problem 15

First we show that $Span\{u, 0\} = Span\{u\}$:

Proof. Since $\forall x \in Span\{u\}$, $x = \alpha u$ for some $\alpha \in \mathbf{R}$, then

$$x = \alpha u + 0, \tag{6}$$

yielding $x \in Span\{u, 0\}$. So $Span\{u, 0\} \supseteq Span\{u\}$.

On the other hand, $\forall x \in Span\{u, 0\}$, $x = \alpha u + \beta 0$ for some $\alpha, \beta \in \mathbf{R}$, then

$$x = \alpha u + \beta 0 = \alpha u, \tag{7}$$

yielding $x \in Span\{u\}$. So $Span\{u, 0\} \subseteq Span\{u\}$.

Therefore , $Span\{u, 0\} = Span\{u\}$. □

Then in the same way we can show that $Span\{u, 2u\} = Span\{u\}$.