MATH 2111: Additional Explanations and Hints to Week 2 Tutorial

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Abstract

This document is provided as a part of supplemental materials for MATH 2111 Matrix Algebra and Applications (2015 autumn). Although it is written in the hope that it will be useful, nothing contained in this document represents the official views or policies of this course. Comments and suggestions are welcomed to be sent to the author (xweiaf@connect.ust.hk).

1 Problem 5

The augmented matrix can be written as

$$A = \left(\begin{array}{ccccc|c} 1 & 2 & 3 & 2 & 15 & 1 \\ 2 & 4 & -1 & 2 & 8 & 6 \\ 3 & 6 & -1 & 3 & 13 & 8 \end{array}\right). \tag{1}$$

To solve the system, we use ERO's to transform the augmented matrix in to RREF

$$\begin{aligned} A &\xrightarrow{-2r_1+r_2} \left(\begin{array}{ccccc|c} 1 & 2 & 3 & 2 & 15 & 1 \\ 0 & 0 & -7 & -2 & -22 & 4 \\ 3 & 6 & -1 & 3 & 13 & 8 \end{array}\right) \xrightarrow{-3r_1+r_3} \left(\begin{array}{ccccc|c} 1 & 2 & 3 & 2 & 15 & 1 \\ 0 & 0 & -7 & -2 & -22 & 4 \\ 0 & 0 & -10 & -3 & -32 & 5 \end{array}\right) \\ &\xrightarrow{-r_3+r_2} \left(\begin{array}{ccccc|c} 1 & 2 & 3 & 2 & 15 & 1 \\ 0 & 0 & 3 & 1 & 10 & -1 \\ 0 & 0 & -10 & -3 & -32 & 5 \end{array}\right) \xrightarrow{\frac{10}{3}r_2+r_3} \left(\begin{array}{ccccc|c} 1 & 2 & 3 & 2 & 15 & 1 \\ 0 & 0 & 3 & 1 & 10 & -1 \\ 0 & 0 & 0 & 1/3 & 4/3 & 5/3 \end{array}\right) \\ &\xrightarrow{r_2/3, r_3\times 3} \left(\begin{array}{ccccc|c} 1 & 2 & 3 & 2 & 15 & 1 \\ 0 & 0 & 1 & 1/3 & 10/3 & -1/3 \\ 0 & 0 & 0 & 1 & 4 & 5 \end{array}\right) (REF) \xrightarrow{-3r_2+r_1} \left(\begin{array}{ccccc|c} 1 & 2 & 0 & 1 & 5 & 2 \\ 0 & 0 & 1 & 1/3 & 10/3 & -1/3 \\ 0 & 0 & 0 & 1 & 4 & 5 \end{array}\right) \\ &\xrightarrow{-r_3+r_1, -r_3/3+r_2} \left(\begin{array}{ccccc|c} 1 & 2 & 0 & 0 & 1 & -3 \\ 0 & 0 & 1 & 0 & 2 & -2 \\ 0 & 0 & 0 & 1 & 4 & 5 \end{array}\right) (RREF) \end{aligned} \tag{2}$$

Now, we can tell that x_1, x_3, x_4 are basic variables, and the solution can be

written as

$$\begin{cases} x_1 = & -3 - 2s - t, \\ x_2 = & s, \\ x_3 = & -2 - 2t, \\ x_4 = & 5 - 4t, \\ x_5 = & t. \end{cases} \tag{3}$$